

## Tutorial Session 6 - Confidence Interval for $\mu$

Q94.3 - Academic Procrastination (P.482)

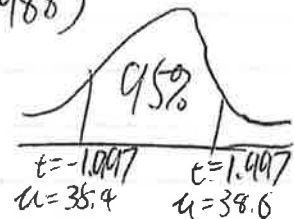
a). Given  $n=65$ ,  $\bar{x}=37$ ,  $s=6.44$ ,  $CI=95\%$ ,  $\alpha/2 = \frac{5\%}{2} = 0.025$   
 $\because \sigma$  is unknown  $\Rightarrow t$  distribution,  $df = n-1 = 65-1 = 64$   
 Critical value  $t_{64}$  with 95% CI  $\Rightarrow 1.997$  (

$$\mu = \bar{x} \pm t_{df} \left( \frac{s}{\sqrt{n}} \right)$$

$$= 37 \pm 1.997 \left( \frac{6.44}{\sqrt{65}} \right) \approx 37 \pm (1.997)(0.7988)$$

$$\approx 37 \pm 1.6$$

$$\therefore 35.4 \leq \mu \leq 38.6$$



We are 95% confident that the mean procrastination scale for 1<sup>st</sup> year students at this college is between 35.4 and 38.6

b). 40 is not contained within 95% confidence interval. Therefore, on average, students at this college do not have high levels of procrastination.

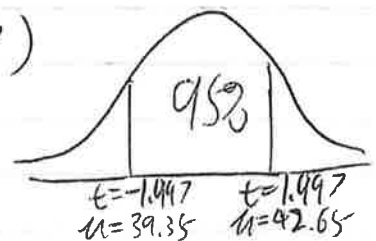
Q94.4 Cont' Q94.3.

a). Given  $n=68$ ,  $\bar{x}=41$ ,  $s=6.82$ ,  $CI=95\%$ ,  $\alpha/2 = \frac{5\%}{2} = 0.025$   
 $\because \sigma$  is unknown  $\Rightarrow t$  distribution,  $df = 68-1 = 67$   
 Critical value  $t_{67}$  with 95% CI  $\Rightarrow 1.997$

$$\mu = 41 \pm 1.997 \left( \frac{6.82}{\sqrt{68}} \right) \approx 41 \pm (1.997)(0.827)$$

$$\approx 41 \pm 1.65$$

$$\therefore 39.35 \leq \mu \leq 42.65$$



We are 95% confident that the mean procrastination scale for 2<sup>nd</sup> year students at this college is between 39.35 and 42.65

b). 2<sup>nd</sup> year students has higher mean procrastination scale than 1<sup>st</sup> year students. It seems, 2<sup>nd</sup> year students tend to procrastinate more than 1<sup>st</sup> year students.

(P.1)

Extra Question 1. — Exercise time

Given  $n=100$ ,  $\sigma=5$ ,  $\bar{x}=15$ ,  $CI=99\%$ ,  $\alpha=1-CI=1-99\%=1\%$

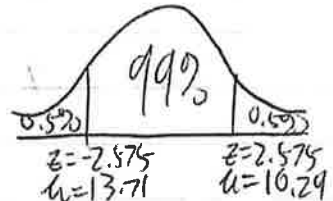
$\therefore \sigma$  is known  $\Rightarrow z$  score  $\Rightarrow \alpha/2 = \frac{0.01}{2} = 0.005$

$$\therefore z_{\alpha/2} = -2.575$$

$$\mu = \bar{x} \pm z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right) = 15 \pm 2.575 \left( \frac{5}{\sqrt{100}} \right) \approx 15 \pm (2.575)(0.5)$$

$$\approx 15 \pm 1.29$$

$$13.71 \leq \mu \leq 16.29$$



$\therefore$  We are 99% confidence that the mean hours spent on doing workshop exercise is between 13.71 and 16.29.

Q9.42 — Medical Research (P.482)

Based on the sample data

$$\sum x_i = 622, n=24, \sum (x_i - \bar{x})^2 = 87.83, CI=90\%$$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{622}{24} \approx 25.92, s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{(n-1)}} \approx \sqrt{\frac{87.83}{24-1}} \approx \sqrt{3.62} \approx 1.95$$

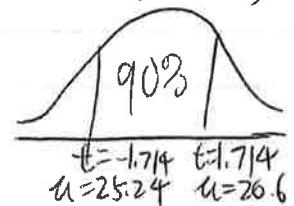
$\therefore \sigma$  is unknown  $\Rightarrow t$  distribution,  $df = 24 - 1 = 23$

Critical value  $t_{23}$  with 90% CI,  $\alpha/2 = \frac{0.1}{2} = 0.05 \Rightarrow 1.714$

$$\mu = \bar{x} \pm t_{df} \left( \frac{s}{\sqrt{n}} \right) \approx 25.92 \pm 1.714 \left( \frac{1.95}{\sqrt{24}} \right) \approx 25.92 \pm (1.714)(0.4)$$

$$\approx 25.92 \pm 0.68$$

$$\therefore 25.24 \leq \mu \leq 26.60$$



$\therefore$  We are 90% confidence that the mean wrist extension is between 25.24 and 26.60.