Univariate Statistical Analysis

Lecture 8

Hypothesis Testing (one-tailed)
Chapter 10
Section 10.4

Today

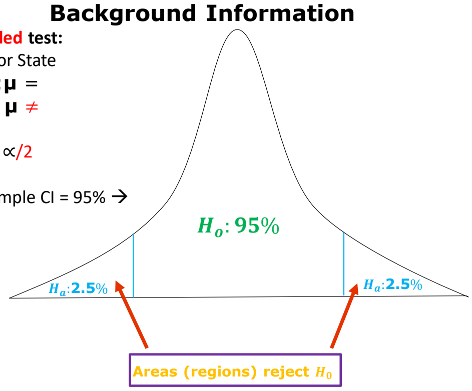
- Hypothesis Test for Population Mean (One Tailed)
  - z test
  - t test

Background Information

Two tailed test:
Define or State
H_0: μ =
H_a: μ ≠

And ∝/2

For example CI = 95% →

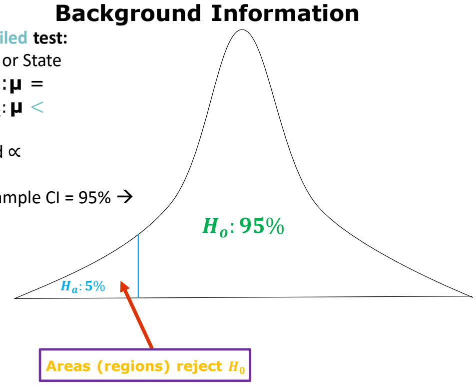


Background Information

One tailed test:
Define or State
H_0: μ =
H_a: μ <

And ∝

For example CI = 95% →

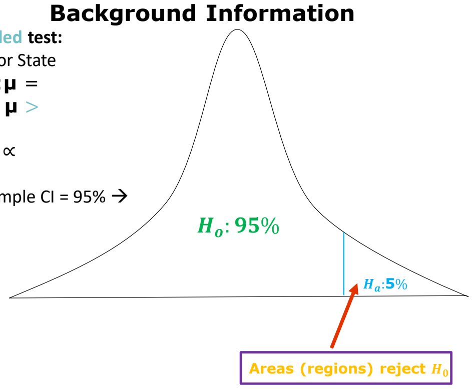


Background Information

One tailed test:
Define or State
H_0: μ =
H_a: μ >

And ∝

For example CI = 95% →



Hypotheses Test Population Mean

Steps
1. Define or State H_0 and H_a.
2. Decide one tail or two tails test.
3. Sketch a normal curve.
4. Find the critical value for z or t.
5. Label step 4. result on the curve.
6. Find the test statistic (value)
7. Label step 6. result on the curve.
8. Decide to reject H_0 or failed to reject H_0.

Example 1

The average value is 70, a researcher suspects that the average value is below 70. Given sample mean of 65, sample size of 100, population standard deviation of 10 and the level of significant is 5%.

Summarize the data:

μ = Population Mean = 70

$\bar{x}$ = Sample Mean = 65

σ = Population Standard Deviation = 10 (z-test)

n = Number of Samples = 100

∝ = Significant Level = 0.05

CI = $1 - \propto = 1 - 0.05 = 0.95 = 95\%$

Example 1 Cont'

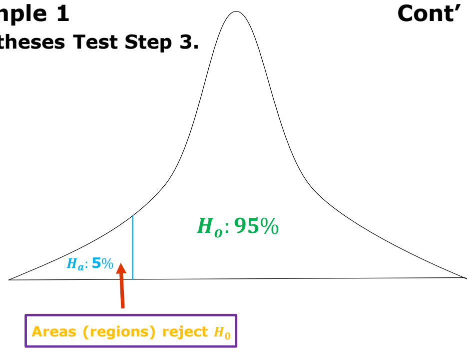
Hypotheses Test Step 1. and 2.

H_0 : μ = 70

H_a : μ < 70

Example 1 Cont'

Hypotheses Test Step 3.



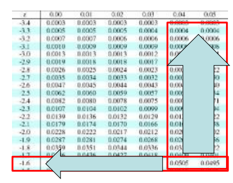
Example 1 Cont'

Hypotheses Test Step 4.

Critical value $z_{\propto}$ (One tailed)

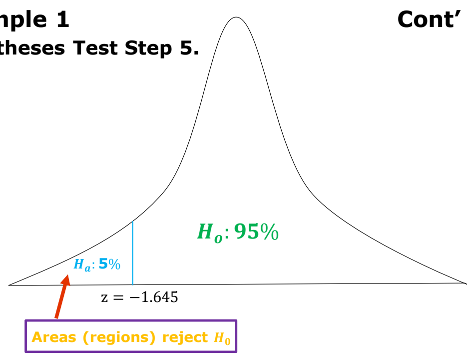
$z_{0.05} = -1.645$

So, critical value is -1.645



Example 1 Cont'

Hypotheses Test Step 5.



Example 1 Cont'

Hypotheses Test Step 6.

Test value

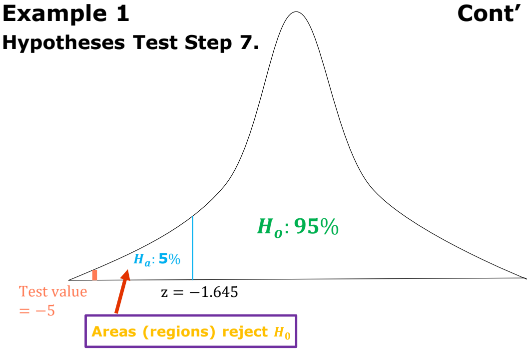
$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{65 - 70}{\frac{10}{\sqrt{100}}}$$

$$= \frac{-5}{1}$$

$$= -5$$

Example 1 Cont’

Hypotheses Test Step 7.



Example 1 Cont’

Hypotheses Test Step 8.

Conclusion:
Reject H_o. There is **enough evidence** to conclude that the average value is less than 70 (H_a).

Example 2

A machine that produces ball with the average diameter 18.53cm, we want to test the average diameter is greater than 18.53cm. Given sample mean of 19.13cm, sample size of 25, population standard deviation of 3.39cm and the level of significant is 0.05.

Summarize the data:

μ = Population Mean = 18.53cm

$\bar{x}$ = Sample Mean = 19.13cm

σ = Population Standard Deviation = 3.39cm (z-test)

n = Number of Samples = 25

$\propto$ = Significant Level = 0.05

CI $= 1 - \propto = 1 - 0.05 = 0.95 = 95\%$

Example 2 Cont’

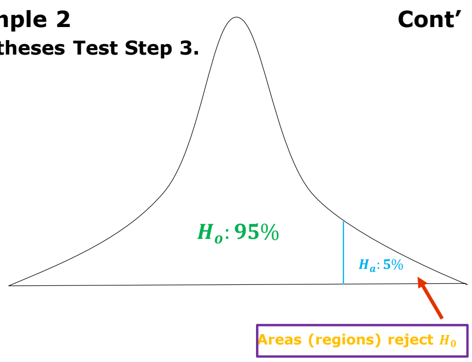
Hypotheses Test Step 1. and 2.

H_0 : μ = 18.53

H_a : μ > 18.53

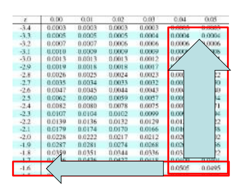
Example 2 Cont’

Hypotheses Test Step 3.



Example 2 Cont’

Hypotheses Test Step 4.



Critical value $z_{\propto}$ (One tailed)

$z_{0.05} = -1.645$

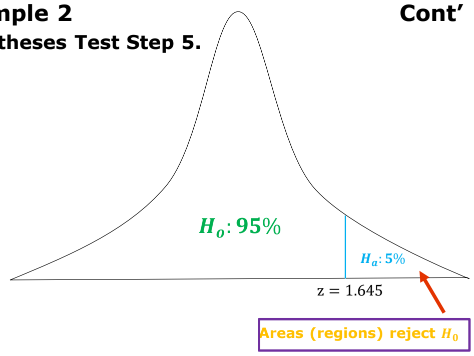
Critical value is $+1.645$

Why the value is not -1.645?
- Normal curve is symmetric → i.e. given $\propto = 0.05$, the z values can be either ± 1.645.

- The z value is on the RIGHT hand side → Positive 1.645

Example 2 — Cont’

Hypotheses Test Step 5.



Example 2 — Cont’

Hypotheses Test Step 6.

Test value

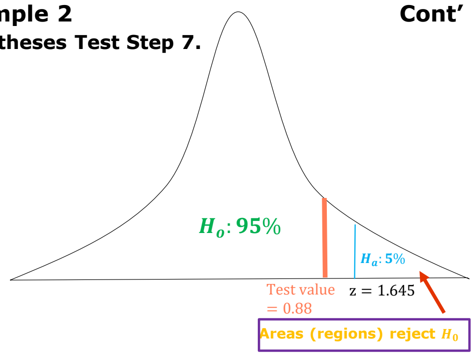
$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{19.13 - 18.53}{\frac{3.39}{\sqrt{25}}}$$

$$= \frac{0.6}{0.678}$$

$$= 0.88$$

Example 2 — Cont’

Hypotheses Test Step 7.



Example 2 — Cont’

Hypotheses Test Step 8.

Conclusion:
Fail to Reject H_o. There is **insufficient evidence** to conclude that the average diameter is greater than 18.53cm(H_a).

Example 3

Given information $\bar{x}$ = 16, s = 5 and n =25, test the claim that the true population mean is greater than 12 at the 0.01 level of significance.

Example 3 — Cont’

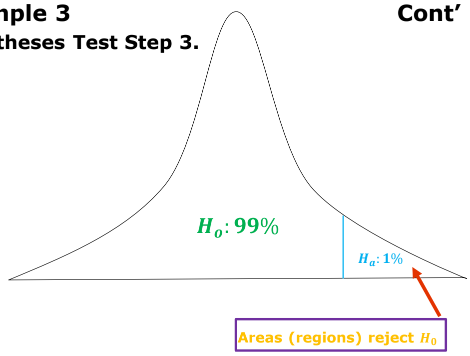
Hypotheses Test Step 1. and 2.

H_0 : μ = 12

H_a : μ > 12

Example 3 **Cont’**

Hypotheses Test Step 3.



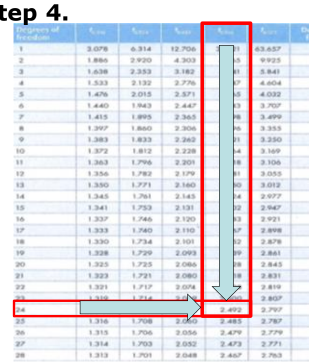
Example 3 **Cont’**

Hypotheses Test Step 4.

Critical t value

$\alpha = 0.01,$
$df = n - 1 = 25 - 1 = 24$

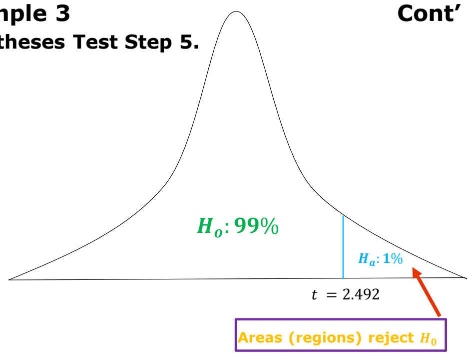
So, critical value is 2.492



Degrees of freedom	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	3.078	6.314	12.706	[illegible]	63.657
2	1.886	2.920	4.303	[illegible]	9.925
3	1.638	2.353	3.182	[illegible]	5.841
4	1.533	2.132	2.776	[illegible]	4.604
5	1.476	2.015	2.571	[illegible]	4.032
6	1.440	1.943	2.447	[illegible]	3.707
7	1.415	1.895	2.365	[illegible]	3.499
8	1.397	1.860	2.306	[illegible]	3.355
9	1.383	1.833	2.262	[illegible]	3.250
10	1.372	1.812	2.228	[illegible]	3.169
11	1.363	1.796	2.201	[illegible]	3.106
12	1.356	1.782	2.179	[illegible]	3.055
13	1.350	1.771	2.160	[illegible]	3.012
14	1.345	1.761	2.145	[illegible]	2.977
15	1.341	1.753	2.131	[illegible]	2.947
16	1.337	1.746	2.120	[illegible]	2.921
17	1.333	1.740	2.110	[illegible]	2.898
18	1.330	1.734	2.101	[illegible]	2.878
19	1.328	1.729	2.093	[illegible]	2.861
20	1.325	1.725	2.086	[illegible]	2.845
21	1.323	1.721	2.080	[illegible]	2.831
22	1.321	1.717	2.074	[illegible]	2.819
23	[illegible]	[illegible]	[illegible]	[illegible]	2.807
24	[illegible]	[illegible]	[illegible]	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763

Example 3 **Cont’**

Hypotheses Test Step 5.



Example 3 **Cont’**

Hypotheses Test Step 6.

Test value

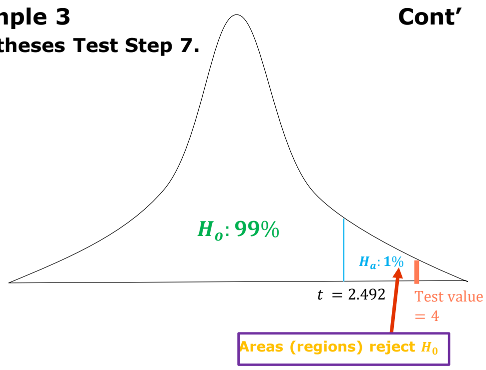
$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{16 - 12}{\frac{5}{\sqrt{25}}}$$

$$= \frac{4}{1}$$

$$= 4$$

Example 3 **Cont’**

Hypotheses Test Step 7.



Example 3 **Cont’**

Hypotheses Test Step 8.

Conclusion:
Reject H_0. There is **enough evidence** to conclude that the true population mean is greater than 12 (H_a).

Example 4

An article claimed that the mean shopping time at a local supermarket was 30 minutes. Given sample mean of 28 minutes, standard deviation of 6 minutes and sample of 9 shoppers. Using 5% level of significance, can we conclude that mean shopping time is less than 30 minutes?

Summarize the data:

μ = Population Mean = 30 minutes

$\bar{x}$ = Sample Mean = 28 minutes

s = Sample Standard Deviation =6 minutes (t-test)

n = Number of Samples = 9

$\propto$ = Significant Level = 0.05

CI $= 1 - \propto = 1 - 0.05 = 0.95 = 95\%$

Example 4 Cont'

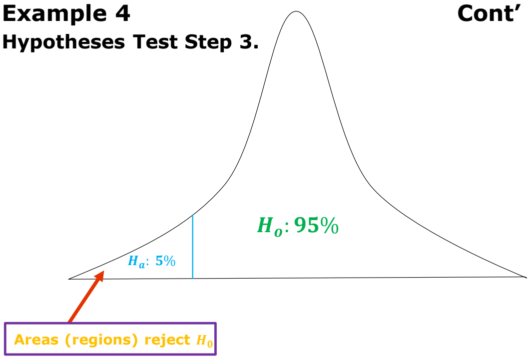
Hypotheses Test Step 1. and 2.

$H_0 : \mu = 30$

$H_a : \mu < 30$

Example 4 Cont'

Hypotheses Test Step 3.



Example 4 Cont'

Hypotheses Test Step 4.

Degrees of freedom	$t_{0.100}$	$t_{0.050}$	$t_{0.025}$	$t_{0.010}$	$t_{0.005}$
1	3.078	[illegible]	12.706	31.821	63.657
2	1.886	[illegible]	4.303	6.965	9.925
3	1.638	[illegible]	3.182	4.541	5.841
4	1.533	[illegible]	2.776	3.747	4.604
5	1.476	[illegible]	2.571	3.365	4.032
6	1.440	[illegible]	2.447	3.143	3.707
7	1.415	[illegible]	2.365	2.998	3.499
8		1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055

Critical t value

$\alpha = 0.05,$
$df = n - 1 = 9 - 1 = 8$

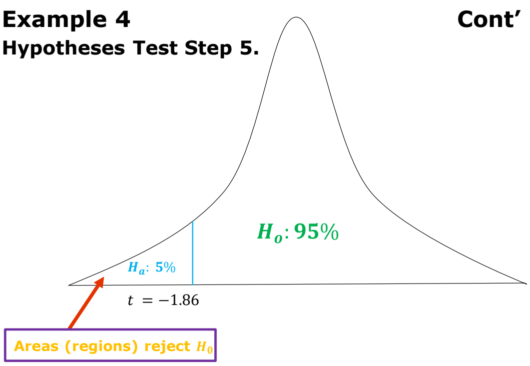
So, critical value is -1.86

$-$ Normal curve is symmetric → i.e. given $\propto = 0.05$, the t values can be either ± 1.86.

$-$ The t value is on the LEFT hand side , i.e. -1.86

Example 4 Cont'

Hypotheses Test Step 5.



Example 4 Cont'

Hypotheses Test Step 6.

Test value

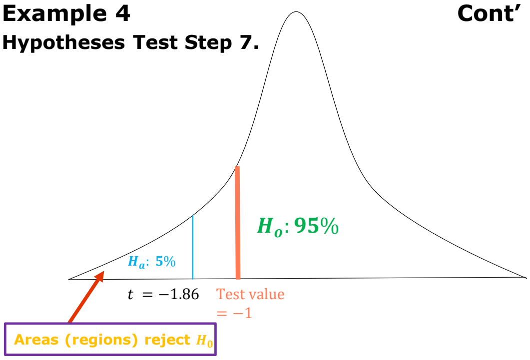
$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{28 - 30}{\frac{6}{\sqrt{9}}}$$

$$= \frac{-2}{2}$$

$$= -1$$

Example 4 **Cont'**

Hypotheses Test Step 7.



Example 4 **Cont'**

Hypotheses Test Step 8.

Conclusion:
Fail to Reject H_o. There is **insufficient evidence** to conclude that mean shopping time is less than 30 minutes(H_a).

Conclusion

- Hypotheses and Test Procedures
- Hypothesis Test for Population Mean, (One Tailed, $\propto$)
- Steps

1. Define or State

H_0: μ = OR H_0: μ =

H_a: μ $>$ H_a: μ $<$

2. Decide one tail or two tails test.
   One Tailed, $\propto$

3. Sketch a normal curve.

Conclusion

4. Find the critical value for z or t.
   Table value

5. Label step 4. result on the curve.

6. Find the test statistic (value)
   - z test (σ is known)

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

   - t test (σ is unknown)

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

7. Label step 6. result on the curve.

Conclusion

8. Decide to reject H_0 or failed to reject H_0.
   - **Reject H_o.** There is **enough evidence** to conclude H_a is true.
   - **Fail to Reject H_o.** There is **insufficient evidence** to conclude that H_a is true.