## Univariate Statistical Analysis

## Lecture 3

## Probability II (Chapter 6)

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## Today

- Conditional Probability
  - Two way Table
  - Venn Diagram
- Independent Events

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## Conditional Probability

The probability of an event GIVEN another event occurred. This can be easier to use two-way table to explain

**DEFINITION**

**Conditional probability:** Suppose that $E$ and $F$ are two events with $P(F) > 0$. **The conditional probability of the event $E$ given that the event $F$ has occurred**, denoted by $P(E|F)$, is

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

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## TWO–WAY TABLE

| | A | A' | Total |
|---|---|---|---|
| B | c | b | c+b |
| B' | a | n | a+n |
| Total | c+a | b+n | a+b+c+n |

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### Last week Students' Example

From a survey of 50 college students, a marketing research company found that 10 students owned stereos only, 15 owned cars only, and 20 owned both cars and stereos.

a. Construct a Venn diagram to summarize the data.
b. Construct a two-way table to summarize the data.
c. P(C)
d. P(C ∪ S)
e. P(C ∩ S)
f. P(S|C)

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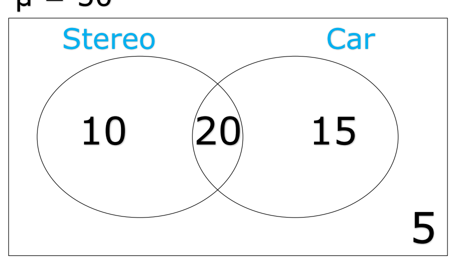
## a. Venn Diagram

Cont'

μ = 50



## b. TWO–WAY TABLE

Cont'

| | Cars | Cars' | Total |
|---|---|---|---|
| Stereos | 20 | 10 | 30 |
| Stereos' | 15 | 5 | 20 |
| Total | 35 | 15 | 50 |

## TWO–WAY TABLE

Cont'

| | Cars | Cars' | Total |
|---|---|---|---|
| Stereos | 20 | 10 | 30 |
| Stereos' | 15 | 5 | 20 |
| Total | 35 | 15 | 50 |

**c.** $P(C) = \frac{20+15}{50} = \frac{35}{50} = 0.7$

## TWO–WAY TABLE

Cont'

| | Cars | Cars' | Total |
|---|---|---|---|
| Stereos | 20 | 10 | 30 |
| Stereos' | 15 | 5 | 20 |
| Total | 35 | 15 | 50 |

**d.** $P(C \cup S) = \frac{20+15+10}{50} = \frac{45}{50} = 0.9$

## TWO–WAY TABLE

Cont'

| | Cars | Cars' | Total |
|---|---|---|---|
| Stereos | 20 | 10 | 30 |
| Stereos' | 15 | 5 | 20 |
| Total | 35 | 15 | 50 |

**e.** $P(C \cap S) = \frac{20}{50} = 0.4$

## TWO–WAY TABLE

Cont'

| | Cars | Cars' | Total |
|---|---|---|---|
| Stereos | 20 | 10 | 30 |
| Stereos' | 15 | 5 | 20 |
| Total | 35 | 15 | 50 |

**f.** $P(S|C) = \frac{P(S \cap C)}{P(C)} = \frac{20}{20+15} = \frac{20}{35} = 0.57$

Or

$P(S|C) = \frac{P(S \cap C)}{P(C)} = \frac{0.4}{0.7} = 0.57$ (from part b. and part d.)

## TWO –WAY TABLE - Probability

| | Cars | Cars' | Total |
|---|---|---|---|
| Stereos | 0.4 | 0.2 | 0.6 |
| Stereos' | 0.3 | 0.1 | 0.4 |
| Total | 0.7 | 0.3 | 1 |

## Example 6.13 (page. 300) – Surviving a Heart Attack (Example 2)

| | | Time to Defibrillation | |
|---|---|---|---|
| Hospital Size | D | D’ | Total |
| S | 1,124 | 576 | 1,700 |
| M | 2,178 | 886 | 3,064 |
| L | 1,387 | 565 | 1,952 |
| Total | 4,689 | 2,027 | 6,716 |

a. P(D)
b. P(D|S)
c. P(D|L)
d. P(L|D)

## Example 2 Cont’
### Convert to percentages Joint Probability

| | | Time to Defibrillation | |
|---|---|---|---|
| Hospital Size | D | D’ | Total |
| S | 0.17 | 0.09 | 0.26 |
| M | 0.32 | 0.13 | 0.45 |
| L | 0.21 | 0.08 | 0.29 |
| Total | 0.7 | 0.3 | 1 |

## Example 2 Cont’

a. P(D) = 0.7
b. P(D|S) = $\frac{0.17}{0.26}$ = 0.65
c. P(D|L) = $\frac{0.21}{0.29}$ = 0.72
d. P(L|D) = $\frac{0.21}{0.7}$ = 0.3

## Independent Events

Independent events mean the outcome from event A **would not affect** the outcome from event B.

E.g. Toss a coin,
Event A: Tail for the first flip;
Event B: Tail for the second flip.

To **show independent** events
**P(A|B)=P(A) or P(B|A)=P(B),**
Otherwise the events are dependents.

## Independent Events

The other way to show independent events, will apply it for next week.

**Multiplication Rule for Two Independent Events**

If the events $E$ and $F$ are independent,

$$P(E \cap F) = P(E)P(F)$$

## Independent Events

Example 3 – Gender and job Promotion
P: a manager is promoted
P’: a manager is not promoted
M: a manager is male
M’: a manager is female
Are the events independent? Justify your answer.

| | P | P’ | Total |
|---|---|---|---|
| M | 46 | 184 | 230 |
| M’ | 8 | 32 | 40 |
| Total | 54 | 216 | 270 |

**Independent Events**

Example 3 – Gender and job Promotion

| | P | P' | Total |
|---|---|---|---|
| M | 0.17 | 0.68 | 0.85 |
| M' | 0.03 | 0.12 | 0.15 |
| Total | 0.2 | 0.8 | 1 |

P(P) = 0.2

P(P|M) = $\frac{0.17}{0.85}$ = 0.2

Or

P(M) = 0.85

P(M|P) = $\frac{0.17}{0.2}$ = 0.85

**Independent Events**

Example 3 – Gender and job Promotion

| | P | P' | Total |
|---|---|---|---|
| M | 0.17 | 0.68 | 0.85 |
| M' | 0.03 | 0.12 | 0.15 |
| Total | 0.2 | 0.8 | 1 |

**Given P(P|M)=P(P),**
we can conclude two events **are independent**. In application, gender **does not affect** promotion to be a manager.

**Independent Events**

Example 4 – Event As and Bs

| | A1 | A2 | A3 | Total |
|---|---|---|---|---|
| B1 | 0.24 | 0.5 | 0.06 | 0.8 |
| B2 | 0.06 | 0.1 | 0.04 | 0.2 |
| Total | 0.3 | 0.46 | 0.1 | 1 |

a. Determine whether the event A1 and event B1 are independent events.

b. Determine whether the event A2 and event B1 are independent events.

**Independent Events**

Example 4 – Event As and Bs Cont'

| | A1 | A2 | A3 | Total |
|---|---|---|---|---|
| B1 | 0.24 | 0.5 | 0.06 | 0.8 |
| B2 | 0.06 | 0.1 | 0.04 | 0.2 |
| Total | 0.3 | 0.46 | 0.1 | 1 |

a. Determine whether the event A1 and event B1 are independent events.

P(A1) = 0.3

P(A1|B1) = $\frac{0.24}{0.8}$ = 0.3

**Given P(A1|B1)=P(A1),**
we can conclude two events **are independent**.

**Independent Events**

Example 4 – Event As and Bs Cont'

| | A1 | A2 | A3 | Total |
|---|---|---|---|---|
| B1 | 0.24 | 0.5 | 0.06 | 0.8 |
| B2 | 0.06 | 0.1 | 0.04 | 0.2 |
| Total | 0.3 | 0.46 | 0.1 | 1 |

b. Determine whether the event A2 and event B1 are independent events.

P(A2) = 0.46

P(A2|B1) = $\frac{0.5}{0.8}$ = 0.63

**Given P(A2|B1)≠P(A2),**
we can conclude two events **are NOT independent**.

## Conclusion

- Conditional Probability
  - Two way Table
  - Venn Diagram
- Independent Events