SHERIDAN
INSTITUTE OF HIGHER EDUCATION

UNIVARIATE STATISTICAL ANALYSIS

TUTORIAL SESSION 5

CONTINUOUS PROBABILITY DISTRIBUTION
NORMAL DISTRIBUTION

Chapter 7

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Then

$$\begin{aligned} x^* &= \mu + z^*\sigma \\ &= 13 + 1.28(3.9) \\ &= 13 + 4.992 \\ &= 17.992 \end{aligned}$$

Interpret the results ➤ About 10% of the garbage trucks using this facility would have a total processing time of more than 17.992 minutes.

The 5% with the fastest processing times would be those with z values less than $z^* = -1.645$ (from Appendix Table 2, based on a cumulative area of 0.05). Then

$$\begin{aligned} x^* &= \mu + z^*\sigma \\ &= 13 + (-1.645)(3.9) \\ &= 13 - 6.416 \\ &= 6.584 \end{aligned}$$

About 5% of the garbage trucks processed at this facility will have total processing times of less than 6.584 minutes.

EXERCISES 7.70 - 7.96

7.70 Determine the following standard normal (z) curve areas. (Hint: See Examples 7.23 and 7.24.)
- **a.** The area under the z curve to the left of 1.75
- **b.** The area under the z curve to the left of -0.68
- **c.** The area under the z curve to the right of 1.20
- **d.** The area under the z curve to the right of -2.82

7.71 Determine the following standard normal (z) curve areas.
- **a.** The area under the z curve between -2.22 and 0.53
- **b.** The area under the z curve between -1 and 1
- **c.** The area under the z curve between -4 and 4

7.72 Determine each of the following areas under the standard normal (z) curve:
- **a.** To the left of -1.28
- **b.** To the right of 1.28
- **c.** Between -1 and 2

7.73 Determine each of the following areas under the standard normal (z) curve:
- **a.** To the right of 0
- **b.** To the right of -5
- **c.** Between -1.6 and 2.5
- **d.** To the left of 0.23

7.74 Let z denote a random variable that has a standard normal distribution. Determine each of the following probabilities:
- **a.** $P(z < 2.36)$
- **b.** $P(z \leq 2.36)$
- **c.** $P(z < -1.23)$
- **d.** $P(1.14 < z < 3.35)$

7.75 Let z denote a random variable that has a standard normal distribution. Determine each of the following probabilities:
- **a.** $P(-0.77 \leq z \leq -0.55)$
- **b.** $P(z > 2)$
- **c.** $P(z \geq -3.38)$
- **d.** $P(z < 4.98)$

7.76 Let z denote a random variable having a normal distribution with $\mu = 0$ and $\sigma = 1$. Determine each of the following probabilities. (Hint: See Examples 7.27 and 7.28.)
- **a.** $P(z < 0.10)$
- **b.** $P(z < -0.10)$
- **c.** $P(0.40 < z < 0.85)$

7.77 Let z denote a random variable having a normal distribution with $\mu = 0$ and $\sigma = 1$. Determine each of the following probabilities.
- **a.** $P(-0.85 < z < -0.40)$
- **b.** $P(-0.40 < z < 0.85)$
- **c.** $P(z > -1.25)$
- **d.** $P(z < -1.50 \text{ or } z > 2.50)$

7.78 Let z denote a variable that has a standard normal distribution. Determine the value z^* to satisfy the following conditions. (Hint: See Example 7.25.)
- **a.** $P(z < z^*) = 0.025$
- **b.** $P(z < z^*) = 0.01$
- **c.** $P(z < z^*) = 0.05$
- **d.** $P(z > z^*) = 0.02$
- **e.** $P(z > z^*) = 0.01$
- **f.** $P(z > z^* \text{ or } z < -z^*) = 0.20$

7.79 Determine the value z^* that

a. Separates the largest 3% of all z values from the others (Hint: See Example 7.26.)

b. Separates the largest 1% of all z values from the others

c. Separates the smallest 4% of all z values from the others

d. Separates the smallest 10% of all z values from the others

7.80 Determine the value of z^* such that

a. $-z^*$ and z^* separate the middle 95% of all z values from the most extreme 5% (Hint: See Example 7.26.)

b. $-z^*$ and z^* separate the middle 90% of all z values from the most extreme 10%

c. $-z^*$ and z^* separate the middle 98% of all z values from the most extreme 2%

d. $-z^*$ and z^* separate the middle 92% of all z values from the most extreme 8%

7.81 Because $P(z < 0.44) = 0.67$, 67% of all z values are less than 0.44, and 0.44 is the 67th percentile of the standard normal distribution. Determine the value of each of the following percentiles for the standard normal distribution. (Hint: If the cumulative area that you must look for does not appear in the z table, use the closest entry, see Example 7.27.)

a. The 91st percentile (Hint: Look for area 0.9100.)

b. The 77th percentile

c. The 50th percentile

d. The 9th percentile

e. What is the relationship between the 70th z percentile and the 30th z percentile?

7.82 Consider the population of all 1-gallon cans of dusty rose paint manufactured by a particular paint company. Suppose that a normal distribution with mean $\mu = 5$ ml and standard deviation $\sigma = 0.2$ ml is a reasonable model for the distribution of the variable $x =$ amount of red dye in the paint mixture. Use the normal distribution model to calculate the following probabilities. (Hint: See Examples 7.27 and 7.28.)

a. $P(x < 5.0)$ **b.** $P(x < 5.4)$

c. $P(x \leq 5.4)$ **d.** $P(4.6 < x < 5.2)$

e. $P(x > 4.5)$ **f.** $P(x > 4.0)$

7.83 Consider babies born in the "normal" range of 37–43 weeks gestational age. The paper referenced in Example 7.27 "Birth Weight Curves Tailored to Maternal World Region" (*Journal of Obstetrics and Gynaecology Canada* [2012]: 159–171) suggests that a normal distribution with mean $\mu = 3500$ grams and standard deviation $\sigma = 600$ grams is a reasonable model for the probability distribution of the variable $x =$ birth weight of a randomly selected full-term baby born in Canada.

a. What is the probability that the birth weight of a randomly selected full-term baby born in Canada exceeds 4000 g?

b. What is the probability that the birth weight of a randomly selected full-term baby born in Canada is between 3000 and 4000 g?

c. What is the probability that the birth weight of a randomly selected full-term baby born in Canada is either less than 2000 g or greater than 5000 g?

7.84 Use the information on birth weights for babies born in Canada given in the previous exercise to answer the following questions.

a. What is the probability that the birth weight of a randomly selected full-term baby born in Canada exceeds 7 pounds? (Hint: 1 lb = 453.59 g.)

b. How would you characterize the most extreme 0.1% of all full-term baby birth weights for babies born in Canada?

c. If x is a random variable with a normal distribution and a is a numerical constant ($a \neq 0$), then $y = ax$ also has a normal distribution. Use this formula to determine the distribution of full-term baby birth weight expressed in pounds (shape, mean, and standard deviation), and then recalculate the probability from Part (a). How does this compare to your previous answer?

7.85 Emissions of nitrogen oxides, which are major constituents of smog, can be modeled using a normal distribution. Let x denote the amount of this pollutant emitted (in parts per billion) by a randomly selected vehicle. Suppose the distribution of x can be described by a normal distribution with $\mu = 1.6$ and $\sigma = 0.4$. A city wants to offer some sort of incentive to get the worst polluters off the road. What emission levels constitute the worst 10% of the vehicles?

7.86 The paper referenced in Example 7.30 ("Estimating Waste Transfer Station Delays Using GPS," *Waste Management* [2008]: 1742–1750) describing processing times for garbage trucks also provided information on processing times at a second facility. At this second facility, the mean total processing time was 9.9 minutes and the standard deviation of the processing times was 6.2 minutes. Explain why a normal distribution with mean 9.9 and standard deviation 6.2 would not be an appropriate model for the probability distribution of the variable $x =$ total processing time of a randomly selected truck entering this facility.

7.87 The size of the left upper chamber of the heart is one measure of cardiovascular health. When the upper