SHERIDAN
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UNIVARIATE STATISTICAL ANALYSIS

TUTORIAL SESSION 2

PROBABILITY I

Chapter 6

Question 6.14 (p.284) – Engineering Construction

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Question 6.35 (p.297) – Student Committee
Only part a to part c.

Question 6.36 (p.297) – Employment Interviews

c. Let C denote the event that the order is for a laptop with a 200 GB hard drive. Are A and C mutually exclusive events? Are B and C mutually exclusive?

6.8 A college library has four copies of a certain book. The copies are numbered 1, 2, 3, and 4. Two of these books are selected at random. The first selected book is placed on 2-hour reserve, and the second book can be checked out overnight.

a. Construct a tree diagram to display the 12 outcomes in the sample space.

b. Let A denote the event that at least one of the books selected is an even-numbered copy. What outcomes are in A?

c. Suppose that copies 1 and 2 are hardcover books, whereas copies 3 and 4 are softcover books. Let B denote the event that exactly one of the copies selected is a hardcover book. What outcomes are contained in B?

6.9 A library has five copies of a certain textbook on reserve of which two copies (1 and 2) are hardcover books and the other three (3, 4, and 5) are softcover books. A student examines these books in random order, stopping only when a softcover book has been selected.

a. Display the possible outcomes in a tree diagram.

b. What outcomes are contained in the event A, that exactly one book is examined before the chance experiment terminates?

c. What outcomes are contained in the event C, that the chance experiment terminates with the examination of book 5?

6.10 Suppose that, starting at a certain time, batteries coming off an assembly line are examined one by one to see whether they are defective (let D = defective and N = not defective). The chance experiment terminates as soon as a nondefective battery is obtained.

a. Give five possible outcomes for this chance experiment.

b. What can be said about the number of outcomes in the sample space?

c. What outcomes are in the event E, that the number of batteries examined is an even number?

6.11 Refer to the previous exercise and now suppose that the chance experiment terminates only when two nondefective batteries have been obtained.

a. Let A denote the event that at most three batteries must be examined before the chance experiment terminates. What outcomes are contained in A?

b. Let B be the event that exactly four batteries must be examined before the chance experiment terminates. What outcomes are in B?

c. What can be said about the number of possible outcomes for this chance experiment?

6.12 A family consisting of three people—P_1, P_2, and P_3—belongs to a medical clinic that always has a physician at each of stations 1, 2, and 3. During a certain week, each member of the family visits the clinic exactly once and is randomly assigned to a station. One experimental outcome is (1, 2, 1), which means that P_1 is assigned to station 1, P_2 to station 2, and P_3 to station 1.

a. List the 27 possible outcomes. (Hint: First list the nine outcomes in which P_1 goes to station 1, then the nine in which P_1 goes to station 2, and finally the nine in which P_1 goes to station 3; a tree diagram might help.)

b. List all outcomes in the event A, that all three people go to the same station.

c. List all outcomes in the event B, that all three people go to different stations.

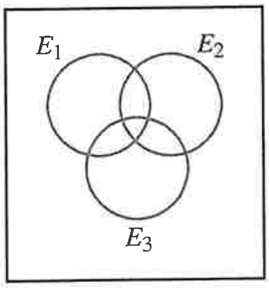
d. List all outcomes in the event C, that no one goes to station 2.

6.13 Using the outcomes for the chance experiment described in the previous exercise, identify outcomes in each of the following events:
a. B^C **b.** C^C **c.** $A \cup B$ **d.** $A \cap B$ **e.** $A \cap C$

6.14 An engineering construction firm is currently working on power plants at three different sites. Define events E_1, E_2, and E_3 as follows:

E_1 = the plant at Site 1 is completed by the contract date
E_2 = the plant at Site 2 is completed by the contract date
E_3 = the plant at Site 3 is completed by the contract date

The following Venn diagram pictures the relationships among these events:



Shade the region in the Venn diagram corresponding to each of the following events. Redraw the Venn diagram for each part of the problem. (Hint: See the discussion of Venn diagrams.)

a. At least one plant is completed by the contract date.

b. All plants are completed by the contract date.
c. None of the plants are completed by the contract date.

6.15 For the events described in the previous exercise, shade the region in the Venn diagram corresponding to each of the following events. Redraw the Venn diagram for each part.
a. Only the plant at Site 1 is completed by the contract date.
b. Exactly one of the three plants is completed by the contract date.
c. Either the plant at Site 1 or both of the other two plants are completed by the contract date.

6.16 Consider a Venn diagram picturing two events A and B that are not mutually exclusive.
a. Shade the event $(A \cup B)^C$. On a separate Venn diagram shade the event $A^C \cap B^C$. How are these two events related?
b. Shade the event $(A \cap B)^C$. On a separate Venn diagram shade the event $A^C \cup B^C$. How are these two events related? (Note: These two relationships together are called DeMorgan's laws.)

SECTION 6.2 Definition of Probability

Reasoning about games of chance has a long history. Archeologists have found evidence that Egyptians used a small bone in mammals, the astragalus, as a sort of four-sided die as early as 3500 B.C. Games of chance were common in Greek and Roman times and during the Renaissance of Western Europe. From this early work related to games of chance, new interpretations of probability have evolved, including an empirical approach preferred by many statisticians today. We begin our discussion of probability with a look at some of the different approaches to probability: the classical, relative frequency, and subjective approaches.

Classical Approach to Probability

Early mathematicians' development of the theory of probability was primarily in the context of gambling and games of chance. A characteristic common to most games of chance is a physical device used to produce outcomes. For example, in children's games, dice or a "spinner" might be used to create chance outcomes.

Dice are constructed so that the physical characteristics of each face are alike except for the number of dots. This ensures that the different outcomes for an individual die are equally likely. If this is the case, the probability of getting a five when a single six-sided die is rolled is one-sixth (1 chance in 6). In general, if there are N *equally likely* outcomes in the sample space for a chance experiment (such as rolling a die), the probability of each of the outcomes is $1/N$.

Classical Approach to Probability for Equally Likely Outcomes

When the outcomes in the sample space of a chance experiment are equally likely, the **probability of an event E**, denoted by $P(E)$, is the ratio of the number of outcomes favorable to E to the total number of outcomes in the sample space:

$$P(E) = \frac{\text{Number of outcomes favorable to } E}{\text{Number of outcomes in the sample space}}$$

According to this definition, the calculation of a probability consists of counting the number of outcomes that make up an event, counting the number of outcomes in the sample space, and then dividing.

Chance experiments that involve tossing fair coins, rolling fair dice, or selecting cards from a well-mixed deck of playing cards have equally likely outcomes. For example, if a fair die is rolled once, each outcome (simple event) has probability 1/6. With E denoting

c. Using the result of Part (b), what is the probability that a hand contains cards from exactly two suits?

6.33 After all students have left the classroom, a statistics professor notices that four copies of the text were left under desks. At the beginning of the next class, the professor distributes the four books at random to the four students (1, 2, 3, and 4) who said they left books. One possible outcome is that 1 receives 2's book, 2 receives 4's book, 3 receives his or her own book, and 4 receives 1's book. This outcome can be abbreviated (2, 4, 3, 1).

a. List the 23 other possible outcomes.

b. Which outcomes are contained in the event that exactly two of the books are returned to their correct owners? Assuming equally likely outcomes, what is the probability of this event?

6.34 Use the information given in the previous exercise to answer the following questions.

a. What is the probability that exactly one of the four students receives his or her own book?

b. What is the probability that exactly three receive their own books?

c. What is the probability that at least two of the four students receive their own books?

6.35 The student council for a school of science and math has one representative from each of the five academic departments: biology (B), chemistry (C), mathematics (M), physics (P), and statistics (S). Two of these students are to be randomly selected for a university-wide student committee. The two students will be selected by placing five slips of paper (numbered 1, 2, 3, 4, and 5) into a bowl, mixing, and drawing out two of them.

a. What are the 10 possible outcomes (simple events)?

b. From the description of the selection process, all simple events are equally likely. What is the probability of each simple event?

c. What is the probability that one of the committee members is the statistics department representative?

d. What is the probability that both committee members come from laboratory science departments?

6.36 A student placement center has requests from five students for interviews for employment with a consulting firm. Three of these students are math majors, and the other two students are statistics majors. Unfortunately, the interviewer only has time to talk with two of the students. These two will be randomly selected from among the five.

a. What is the probability that both selected students are statistics majors?

b. What is the probability that both students selected are math majors?

c. What is the probability that at least one of the students selected is a statistics major?

d. What is the probability that the selected students have different majors?

6.37 Suppose that a six-sided die is weighted so that any even-numbered face is twice as likely to land face up as any odd-numbered face. Consider the chance experiment that consists of rolling this die.

a. What are the probabilities of the six simple events? (Hint: Denote these events by $O_1, \ldots, O_6$. Then $P(O_1) = p$, $P(O_2) = 2p$, $P(O_3) = p, \ldots$, $P(O_6) = 2p$. Now use a condition on the sum of these probabilities to determine p.)

b. What is the probability that the number showing is an odd number? What is the probability that the number showing is at most three?

c. Now suppose that the die is weighted so that the probability of each simple event is proportional to the number showing on the corresponding upturned face; that is, $P(O_1) = c$, $P(O_2) = 2c, \ldots$, $P(O_6) = 6c$. What are the probabilities of the six simple events? Calculate the probabilities of Part (b) for this die.

SECTION 6.4 Conditional Probability

Sometimes the knowledge that one event has occurred changes our assessment of the likelihood that another event occurs. For example, consider a population in which 0.1% of all individuals have a certain disease. It is difficult to tell if someone has the disease based on a physical exam, but there is a diagnostic test available. Unfortunately, the test is not always correct. Of those with positive test results, 80% actually have the disease and the other 20% who show positive test results are false-positives.

To put this in probability terms, consider the chance experiment of randomly selecting a person from the population. Define the following events:

E = event that the individual has the disease
F = event that the individual's diagnostic test is positive