The point estimates of β, the slope, and α, the y intercept of the population regression line, are the slope and y intercept of the least-squares line:

$$b = \text{point estimate of } \beta = \frac{S_{xy}}{S_{xx}}$$

$$a = \text{point estimate of } \alpha = \bar{y} - b\bar{x}$$

where

$$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n} \quad \text{and} \quad S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

The estimated population regression line is the least-squares line

$$\hat{y} = a + bx$$

Suppose x^* denotes a specified value of the predictor variable x. Then $a + bx^*$ has two different interpretations:

1. It is a point estimate of the mean y value when $x = x^*$.
2. It is a point prediction of an individual y value to be observed when $x = x^*$.

Example 13.2 Mother's Age and Baby's Birth Weight

Understand the context ➤

● Medical researchers have noted that adolescent females are much more likely to deliver low-birth-weight babies than are adult females. Because low-birth-weight babies have higher mortality rates, a number of studies have examined the relationship between birth weight and mother's age for babies born to young mothers.

One such study is described in the article "Body Size and Intelligence in 6-Year-Olds: Are Offspring of Teenage Mothers at Risk?" (*Maternal and Child Health Journal* [2009]: 847–856). The following data on

x = Maternal age (in years)

and

y = Birth weight of baby (in grams)

are consistent with summary values given in the referenced article and also with data published by the National Center for Health Statistics.

Consider the data ➤

	Observation									
	1	2	3	4	5	6	7	8	9	10
x	15	17	18	15	16	19	17	16	18	19
y	2,289	3,393	3,271	2,648	2,897	3,327	2,970	2,535	3,138	3,573

A scatterplot of the data is given in Figure 13.7. The scatterplot shows a linear pattern, and the variability in the y values appears to be similar across the range of x values. This supports the appropriateness of the simple linear regression model.

Do the work ➤ The summary statistics (calculated from the given sample data) are

$$n = 10 \qquad \Sigma x = 170 \qquad \Sigma y = 30{,}041$$
$$\Sigma x^2 = 2910 \qquad \Sigma xy = 515{,}600 \qquad \Sigma y^2 = 91{,}785{,}351$$

● Data set available online