Univariate Statistical Analysis

Lecture 5

Continuous Probability Distributions
(Chapter 7)

Today

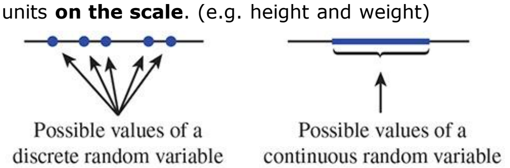
- Continuous probability distribution
  - Normal Distribution
    - Finding the area = probabilities
    - Central Limited Theorem

Review

From Lecture 1

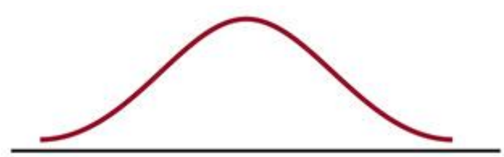
Discrete variable is one in which there are **no possible values between** adjacent units on the **scale**. (e.g. number of students , how many chairs)

Continuous variable is one that theoretically can have an **infinite number** of values between adjacent units **on the scale**. (e.g. height and weight)



Normal Probability Distribution

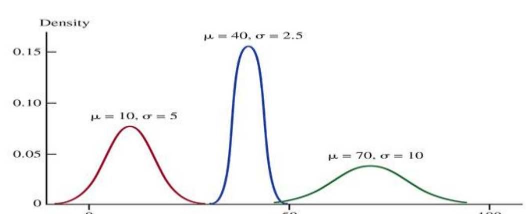
- Continuous
- Bell-shaped
- Symmetric
- Total area under the curve is equal to 1



Normal Probability Distribution

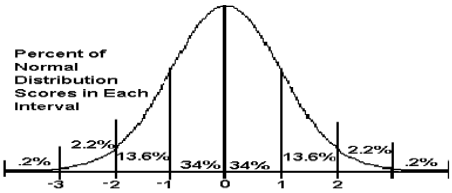
Parameters are
- ***μ***, the mean of a normal distribution describes where the corresponding curve is centered.
- ***σ***, the standard deviation of a normal distribution describes how much the curve spreads out around the center.

Normal Probability Distribution

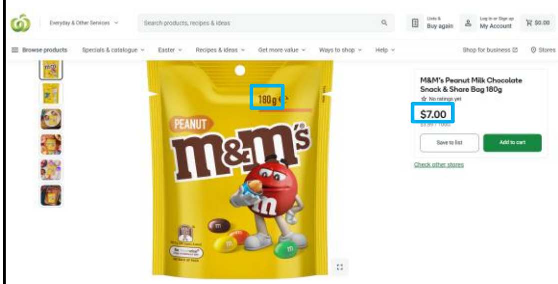


Interpretations are
- Smaller the σ value, the curve is more taller and narrower. Because larger area is concentrated near the center of the curve as the blue curve.
- Higher the σ value, the curve is more shorter and wider. Because small area is concentrated near the center of the curve as the green curve.

Normal Probability Distribution



Normal Probability Distribution



Every pack 180g?

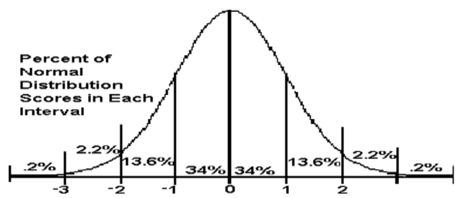
Example 1 - Finding Area

Let $\mu = 0$, $\sigma = 1$

Calculate the followings:

a. $P(x \leq 0)$
b. $P(x \geq 0)$
c. $P(x \geq 1)$
d. $P(x \leq -2)$
e. $P(-1 \leq x < 2)$
f. $P(x > 3)$
g. $P(x = 1)$

Example 1 - Finding Area cont'



a. $P(x \leq 0) = 0.34 + 0.136 + 0.022 + 0.002 = 0.5$
b. $P(x \geq 0) = 0.5$
c. $P(x \geq 1) = 0.136 + 0.022 + 0.002 = 0.16$
d. $P(x \leq -2) = 0.002 + 0.022 = 0.024$
e. $P(-1 \leq x < 2) = 0.34 + 0.34 + 0.136 = 0.816$
f. $P(x > 3) = 0.002$
g. $P(x = 1) = 0$

Example 2 - Finding Area

The scores of a mid-term Accounting test are normally distributed, with $\mu = 60$ and $\sigma = 10$.

Calculate the followings:

a. $P(x \leq 60)$
b. $P(x \leq 50)$
c. $P(x \geq 80)$
d. $P(45 \leq x < 50)$

We can't do this question unless we convert x scores into z scores (i.e. $\mu = 0$ and $\sigma = 1$).

$$z = \frac{x - \mu}{\sigma}$$

HIGHLY recommended to draw a picture of the curve and locate the relevant areas on it before convert to z score.

Example 2 - Finding Area Cont'

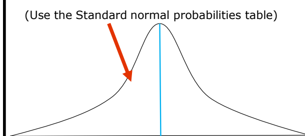
Given $\mu = 60, \sigma = 10$ and $z = \frac{x - \mu}{\sigma}$

a. $P(x \leq 60)$

$$z = \frac{x - \mu}{\sigma} = \frac{60 - 60}{10} = \frac{0}{10} = 0$$

So, $P(x \leq 60) = P(z \leq 0) = 0.5000$

(Use the Standard normal probabilities table)



z	0.00	0.01
-3.4	0.0003	0.0003
-3.3	0.0005	0.0005
-3.2	0.0007	0.0007
-3.1	0.0010	0.0009
-3.0	0.0013	0.0013
-2.9	0.0019	0.0018
-2.8	0.0026	0.0025
-2.7	0.0035	0.0034
-2.6	0.0047	0.0045
-2.5	0.0062	0.0060
-2.4	0.0082	0.0080
-2.3	0.0107	0.0104
-2.2	0.0139	0.0136
-2.1	0.0179	0.0174
-2.0	0.0228	0.0222
-1.9	0.0287	0.0281
-1.8	0.0359	0.0351
-1.7	0.0446	0.0436
-1.6	0.0548	0.0537
-1.5	0.0668	0.0655
-1.4	0.0808	0.0793
-1.3	0.0968	0.0951
-1.2	0.1151	0.1131
-1.1	0.1357	0.1335
-1.0	0.1587	0.1562
-0.9	0.1841	0.1814
-0.8	0.2119	0.2090
-0.7	0.2420	0.2389
-0.6	0.2743	0.2709
-0.5	0.3085	0.3050
-0.4	0.3446	0.3409
-0.3	0.3821	0.3783
-0.2	0.4207	0.4168
[illegible]	[illegible]	0.4562
-0.0	0.5000	0.4960

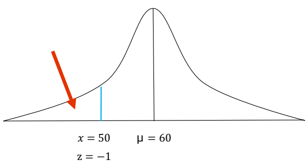
Example 2 - Finding Area Cont'

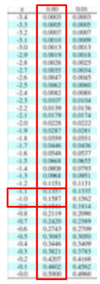
Given $\mu = 60, \sigma = 10$ and $z = \frac{x - \mu}{\sigma}$

b. $P(x \leq 50)$

$$z = \frac{x - \mu}{\sigma} = \frac{50 - 60}{10} = \frac{-10}{10} = -1$$

So, $P(x \leq 50) = P(z \leq -1) = 0.1587$





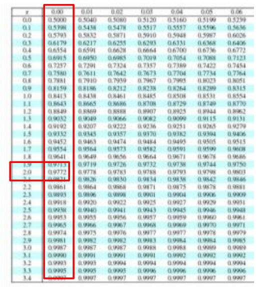
Example 2 - Finding Area Cont'

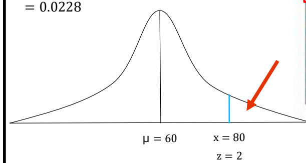
Given $\mu = 60, \sigma = 10$ and $z = \frac{x - \mu}{\sigma}$

c. $P(x \geq 80)$

$$z = \frac{x - \mu}{\sigma} = \frac{80 - 60}{10} = \frac{20}{10} = 2$$

So, $P(x \geq 80) = P(z \geq 2)$
$= 1 - P(z \leq 2)$
$= 1 - 0.9772$
$= 0.0228$





Example 2 - Finding Area Cont'

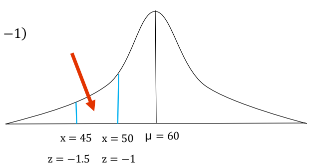
Given $\mu = 60, \sigma = 10$ and $z = \frac{x - \mu}{\sigma}$

d. $P(45 \leq x < 50)$

$$z = \frac{x - \mu}{\sigma} = \frac{50 - 60}{10} = \frac{-10}{10} = -1$$

$$z = \frac{x - \mu}{\sigma} = \frac{45 - 60}{10} = \frac{-15}{10} = -1.5$$

So, $P(45 \leq x < 50) = P(-1.5 \leq z < -1)$
$= P(z < -1) - P(z \leq -1.5)$
$= 0.1587 - 0.0668$
$= 0.0919$



Example 3 - Finding Area

The life of AA brand batteries is normally distributed with an average of 25 hours and a standard deviation of 4 hours. What is the probability that a randomly selected battery can be last for:

a. less than 24 hours
b. longer than 30 hours
c. between 23 and 28 hours

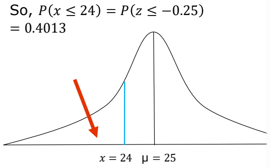
Example 3 - Finding Area Cont'

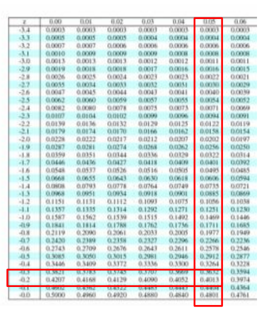
Given $\mu = 25, \sigma = 4$ and $z = \frac{x - \mu}{\sigma}$

a. $P(x \leq 24)$

$$z = \frac{x - \mu}{\sigma} = \frac{24 - 25}{4} = \frac{-1}{4} = -0.25$$

So, $P(x \leq 24) = P(z \leq -0.25)$
$= 0.4013$





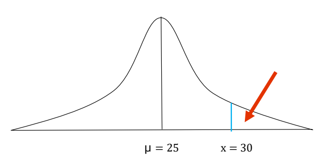
Example 3 - Finding Area Cont'

Given $\mu = 25, \sigma = 4$ and $z = \frac{x - \mu}{\sigma}$

b. $P(x \geq 30)$

$$z = \frac{x - \mu}{\sigma} = \frac{30 - 25}{4} = \frac{5}{4} = 1.25$$

So, $P(x \geq 80) = P(z \geq 1.25)$
$= 1 - P(z \leq 1.25)$
$= 1 - 0.8944$
$= 0.1056$



Example 3 - Finding Area Cont’

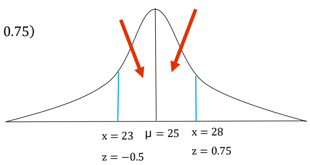
Given $\mu = 25, \sigma = 4$ and $z = \frac{x - \mu}{\sigma}$

$\boldsymbol{d.\ P(23 \le x < 28)}$

$$z = \frac{x - \mu}{\sigma} = \frac{23 - 25}{4} = \frac{-2}{4} = -0.5000$$

$$z = \frac{x - \mu}{\sigma} = \frac{28 - 25}{4} = \frac{3}{4} = 0.7500$$

So, $P(23 \le x < 28) = P(-0.5 \le z < 0.75)$
$= P(z < 0.75) - P(z \le -0.5)$
$= 0.7734 - 0.3085$
$= 0.4649$



Central Limited Theorem

When the sample size is large enough (n>30), the distribution can be approximated by a normal distribution, even if the underlying distribution is not normally distributed.

Conclusion

- Continuous probability distribution
  - Normal Distribution
    - Finding the area = probabilities
    - Central Limited Theorem

Reference

Azzolino, A. (2005). *MIDDLE GROUND - Answers*. Retrieved from http://www.mathnstuff.com/math/spoken/here/2class/90/ans90.htm