Univariate Statistical Analysis

Lecture 2

Probability I (Chapter 6)

Structure of Probability

- Random Experiment
- Random Sample
- Sample Space
- Probability
- Venn Diagram
- Probability Rules

- Random experiment
  - A random experiment is a process by which we observe something uncertain.
  - Outcome is a result of a random experiment.
- Examples

Random Experiment	***Outcomes***
– Playing cards	Within the 52 cards
– Flip a fair coin	Heads and tails
– Record statistics test marks	Numbers between 0 and 100

- Random sample
  - Sample is being equally selected from the population in an random experiment.
- Sample space
  - The set of all possible outcomes.

Example:
Random experiment of toss a coin; sample space (S) = {heads, tails}
We may express the data into Venn diagram, two way table and tree diagram.

Probability

The chance that a particular outcome will occur in a random experiment within the sample space.

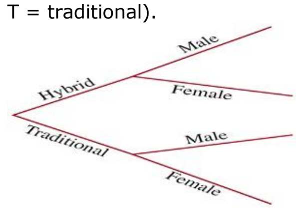
Example 1
In rolling a fair die once, what is the probability of rolling a 3?
S = {1,2,3,4,5,6}
The probability of rolling a 3 is $\frac{1}{6}$. It is because rolling a 3 only occur once out of the 6 possible outcomes.

We can express in this way. $\mathbf{P(3) = \frac{1}{6}}$

Probability

Example 2
Example 6.1 (Page. 280) – Car preferences

A person who purchased a Honda Civic was categorized by sex (M or F) and type of engine purchased (H = hybrid, T = traditional).
S = {HM, HF, TM, TF}



Probability

Example 2
Example 6.1 (Page. 280) – Car preferences (Cont')

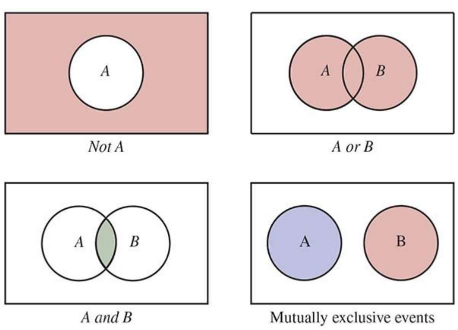
The probability of a Male purchased Honda Civic is $\frac{2}{4}$ or $\frac{1}{2}$. It is because HM and TM out of the 4 possible outcomes.

We can express in this way. **P(M) =** $\mathbf{\frac{1}{2}}$

So, probability of an event happening = $\frac{\textit{Number of ways it can happen}}{\textit{Total number of outcomes}}$

0 ≤ Probability ≤ 1

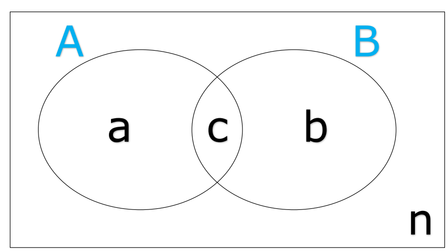
Venn Diagram



Venn Diagram

- Graphic representation of sets

Example: given 2 events A,B;
S = {a,b,c,n}



Venn Diagram

- A or B = A ∪ B = {a,c,b}
- A and B = A ∩ B = {c}
- complement A = A^c = A' = {b,n}
- complement B = B^c = B' = {a,n}
- complement (A or B) = $(A \cup B)^c$ $(A \cup B)'$ = {n}

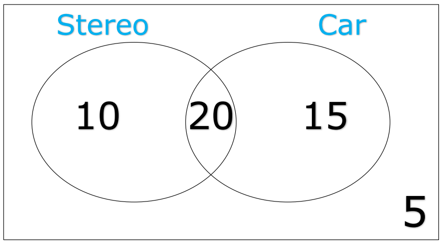
Venn Diagram

Example 3 : From a survey of 50 college students, a marketing research company found that 10 students owned stereos only, 15 owned cars only, and 20 owned both cars and stereos.

a. How many students did not own either a car or a stereo.
b. How many students owned either a car or a stereo.
c. P(car)
d. P(car)'
e. P(car ∪ stereo)

Venn Diagram

$\mu = 50$



Venn Diagram

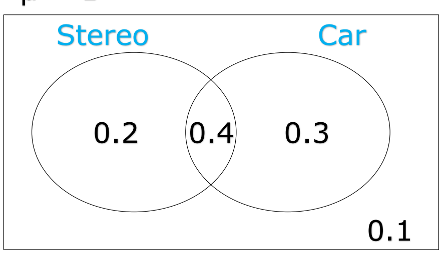
Solution Example 3:

a. How many students did not own either a car or a stereo.
(car ∪ stereo)' = 50-10-20-15 = 5

b. How many students owned either a car or a stereo.
(car ∪ stereo) = 10+20+15 = 45 or ......?

Venn Diagram - Probability

$\mu = 1$



Venn Diagram

Solution Example 3:

c. P(car) = 0.3 + 0.4 = 0.7

d. P(car)' = 0.2 + 0.1 =0.3

Probability Rules

- ❑ Additional Rule: P(A or B) = P(A) + P(B) – P(A and B)
- ❑ Complement Rule: $P(A) = 1- P(\bar{A})$

Revisit example 2d. and 2e.

Venn Diagram

Solution Example 3:

d. P(car)' = 0.2 + 0.1 =0.3
or
P(car)' = 1 – P(car) = 1-0.7 = 0.3

e. P(car ∪ stereo) = P(car) + P(stereo) – P(car ∩ stereo)
= 0.4 + 0.3 + 0.2 + 0.4 – 0.4
= 0.9

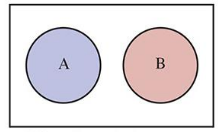
Mutually exclusive

DEFINITION

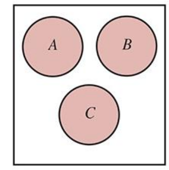
Mutually exclusive: Two events are **mutually exclusive** if they have no outcomes in common. The term **disjoint** is also sometimes used to describe events that have no outcomes in common.

Events are mutually exclusive means that events **cannot occur** at the **same time**.

Mutually exclusive



Mutually exclusive events



e mutually exclusive events

Mutually exclusive

The Addition Rule for Mutually Exclusive Events

Suppose E and F are mutually exclusive events. Then

$$P(E \text{ or } F) = P(E \cup F) = P(E) + P(F)$$

This property of probability is known as the addition rule for mutually exclusive events. More generally, if events $E_1, E_2, \ldots, E_k$ are all mutually exclusive, then

$$P(E_1 \text{ or } E_2 \text{ or} \ldots \text{or } E_k) = P(E_1 \cup E_2 \cup \cdots \cup E_k) = P(E_1) + P(E_2) + \cdots + P(E_k)$$

In words, the probability that any of these k mutually exclusive events occurs is the sum of the probabilities of the individual events.

Also, P(A and B) = 0

Conclusion

- Random Experiment
- Random Sample
- Sample Space
- Probability
- Venn Diagram
- Probability Rules