Univariate Statistical Analysis

Lecture 4

Discrete Probability Distributions (Chapter 7)

Today

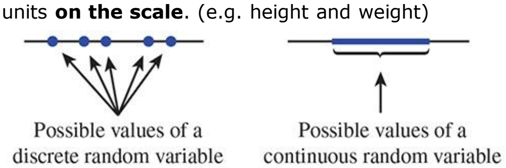
- Discrete probability distributions
  - Binomial Distribution
  - Geometric Distribution

Review

From Lecture 1

Discrete variable is one in which there are **no possible values between** adjacent units on the **scale**. (e.g. number of students , how many chairs)

Continuous variable is one that theoretically can have an **infinite number** of values between adjacent units **on the scale**. (e.g. height and weight)



Review

From Lecture 3
Independent events mean the outcome from event A **would not affect** the outcome from event B.

Multiplication Rule for Two Independent Events

If the events E and F are independent,

$P(E \cap F) = P(E)P(F)$

Probability Distribution Discrete

The probability distribution of a discrete random variable x gives the probability associated with each possible x value.

Common ways to display it are a table, a probability histogram, or a formula.

Example 1

Example 7.5 (page.347) – Energy Efficient Refrigerators

4 customers randomly selected for purchasing a refrigerator
Choose either
(E) energy efficient model
(G) less expensive model
40% of all customers select (E), i.e. P(E)=0.4; P(G) = 1 - 0.4 = 0.6

Due to the customers are independent, one customer decision would not affect the other customers, if we have to find
P(EGGE) = P(E) P(G) P(G) P(E)
= 0.4 x 0.6 x 0.6 x 0.4
= 0.0576

Similarly,

P(EGEG) = P(E) P(G) P(E) P(G)
= 0.4 x 0.6 x 0.4 x 0.6
= 0.0576

Example 1
Example 7.5 (page.347) – Energy Efficient Refrigerators Cont'

$x =$ the number of customers purchase (E) refrigerators, all the possible outcomes are following:

Outcome	Probability	x Value	Outcome	Probability	x Value
GGGG	0.1296	0	GEEG	0.0576	2
EGGG	0.0864	1	GEGE	0.0576	2
GEGG	0.0864	1	GGEE	0.0576	2
GGEG	0.0864	1	GEEE	0.0384	3
GGGE	0.0864	1	EGEE	0.0384	3
EEGG	0.0576	2	EEGE	0.0384	3
EGEG	0.0576	2	EEEG	0.0384	3
EGGE	0.0576	2	EEEE	0.0256	4

Example 1
Example 7.5 (page.347) – Energy Efficient Refrigerators Cont'

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GGEG	0.0864	1	GEEE	0.0384	3
GGGE	0.0864	1	EGEE	0.0384	3
EEGG	0.0576	2	EEGE	0.0384	3
EGEG	0.0576	2	EEEG	0.0384	3
EGGE	0.0576	2	EEEE	0.0256	4

$P(x = 0) = P(GGGG) = 0.1296$
$P(x = 1) = P(EGGG) + P(GEGG) + P(GGEG) + P(GGGE)$
$= 0.0864 + 0.0864 + 0.0864 + 0.0864$
$= 0.3456$

Example 1
Example 7.5 (page.347) – Energy Efficient Refrigerators Cont'

$P(x = 0) = P(GGGG) = 0.1296$

$P(x = 1) = P(EGGG) + P(GEGG) + P(GGEG) + P(GGGE)$
$= 0.0864 + 0.0864 + 0.0864 + 0.0864$
$= 4(0.0864)$
$= 0.3456$

$P(x = 2) = P(EEGG) + P(EGEG) + \cdots + P(GGEE)$
$= 6(0.0576)$
$= 0.3456$

$P(x = 3) = P(EEEG) + P(EGEE) + \cdots + P(GEEE)$
$= 4(0.0384)$
$= 0.1536$

$P(x = 4) = P(EEEE) = 0.0256$

Example 1
Example 7.5 (page.347) – Energy Efficient Refrigerators Cont'

How about
a. $P(x \le 1)$
b. $P(x \ge 2)$
c. $P(x \ge 0)$
d. $P(x \le 4)$
e. $P(x < 4)$
f. $P(x > 4)$

Example 1
Example 7.5 (page.347) – Energy Efficient Refrigerators Cont'

a. $P(x \le 1) = P(x = 0) + P(x = 1)$
$= 0.1296 + 0.3456$
$= 0.4752$

b. $P(x \ge 2) = P(x = 2) + P(x = 3) + P(x = 4)$
$= 0.3456 + 0.1536 + 0.0256$
$= 0.5248$

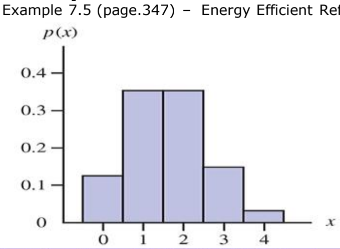
c. $P(x \ge 0) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4)$
$= 0.1296 + 0.3456 + 0.3456 + 0.1536 + 0.0256$
$= 1$

d. $P(x \le 4) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4)$
$= 0.1296 + 0.3456 + 0.3456 + 0.1536 + 0.0256$
$= 1$

e. $P(x < 4) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$
$= 0.1296 + 0.3456 + 0.3456 + 0.1536$
$= 0.9744$

f. $P(x > 4) = 0$

Example 1
Example 7.5 (page.347) – Energy Efficient Refrigerators Cont'



Properties of Discrete Probability Distributions

Properties of Discrete Probability Distributions

1. For every possible x value, $0 \le p(x) \le 1$.
2. $\sum_{\text{all } x \text{ values}} p(x) = 1$

Binomial Distribution

Note that:

- Trial means observation
- "Success" is a name of one of the possible outcomes result.

Properties of a Binomial Experiment

A **binomial experiment** consists of a sequence of trials with the following conditions:

1. There are a fixed number of trials.
2. Each trial can result in one of only two possible outcomes, labeled success (S) and failure (F).
3. Outcomes of different trials are independent.
4. The probability that a trial results in a success is the same for each trial.

The **binomial random variable** x is defined as

x = number of successes observed when a binomial experiment is performed

The probability distribution of x is called the **binomial probability distribution**.

Binomial Distribution

The Binomial Distribution

Notation:

n = number of independent trials in a binomial experiment
p = probability of success for each trial

Then

$$p(x) = P(x \text{ successes among } n \text{ trials})$$

$$= \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x} \quad x = 0, 1, 2, \ldots, n$$

The expressions $\binom{n}{x}$ or ${}_nC_x$ are sometimes used in place of $\frac{n!}{x!(n-x)!}$. Both are read as "n choose x" and represent the number of ways of choosing x items

(continued)

Binomial Distribution

from a set of n. Using this notation, the binomial probability function can also be written as

$$p(x) = \binom{n}{x}p^x(1-p)^{n-x} \quad x = 0, 1, 2, \ldots, n$$

or

$$p(x) = {}_nC_x p^x(1-p)^{n-x} \quad x = 0, 1, 2, \ldots, n$$

Example 2

Example 7.19 (page.375) – Computer Sales

12 customers → n → number of trials
x = number of computers among these 12 that are laptops
60% Laptop computers sold → probability of "success" → $p(S)$ $= P(x = S)$
40% Desktop computers sold → probability of "failure"

Hence, the probability distribution of x is given by

$$p(x) = \frac{12!}{x!\,(12-x)!}(0.6)^x(0.4)^{(12-x)} \quad , \quad x = 0, 1, 2, \ldots \ldots, 12$$

a. What is the probability that exactly four computers are laptops?
b. $P(4 \le x \le 7)$
c. $P(4 < x < 7)$

Example 2

Example 7.19 (page.375) – Computer Sales Cont'

$$p(x) = \frac{12!}{x!\,(12-x)!}(0.6)^x(0.4)^{(12-x)} \quad , \quad x = 0, 1, 2, \ldots \ldots, 12$$

a. What is the probability that exactly four computers are laptops?

$$P(x = 4) = p(4) = \frac{12!}{4!\,(12-4)!}(0.6)^4(0.4)^{(12-4)}$$

$$= \frac{12!}{4!\,(8)!}(0.6)^4(0.4)^{(8)}$$

$$= (495)\,(0.6)^4(0.4)^{(8)}$$

$$= 0.042$$

Example 2

Example 7.19 (page.375) – Computer Sales Cont'

b. $P(4 \le x \le 7) = p(4) + p(5) + p(6) + p(7)$

$$= \frac{12!}{4!(12-4)!}(0.6)^4(0.4)^{(12-4)} + \ldots\ldots + \frac{12!}{7!(12-7)!}(0.6)^4(0.4)^{(12-7)}$$

$$= 0.042 + 0.101 + 0.177 + 0.227$$

$$= 0.547$$

c. $P(4 < x < 7) = p(5) + p(6)$

$$= \frac{12!}{5!\,(12-5)!}(0.6)^4(0.4)^{(12-5)} + \frac{12!}{6!\,(12-6)!}(0.6)^4(0.4)^{(12-6)}$$

$$= 0.101 + 0.177$$

$$= 0.278$$

Table Method
Example 3
Exercise 7.54 (page.381) – FlightView

10 customers → n → number of trials
x = to be the number of the 10 selected passengers who travel with a smart phone
80% passengers who travel with a smart phone → probability of "success" → $p(S) = P(x = S)$

a. $p(8)$
b. $P(x \leq 7)$
c. Calculate the probability that more than half of the selected passengers travel with a smart phone.

Table Method
Example 3
Exercise 7.54 (page.381) – FlightView Cont'
Appendix Table 9. (Page. 802 - 804)

a. $p(8)$
Given $n = 10, p = 0.8$
$P(x = 8) = 0.302$

b. $P(x \leq 7)$
Given $n = 10, p = 0.8$
$P(x \leq 7) = p(0) + p(1) + p(2) + \dots\dots + p(7)$
$= 0.000 + 0.000 + 0.000 + \dots\dots + 0.201$
$= 0.322$

Or

$P(x \leq 7) = 1 - P(x > 7) = 1 - (p(8) + p(9) + p(10))$
$= 1 - (0.302 + 0.268 + 0.107)$
$= 0.323$

Table Method
Example 3
Exercise 7.54 (page.381) – FlightView Cont'
Appendix Table 9. (Page. 802 - 804)

c. Calculate the probability that more than half of the selected passengers travel with a smart phone.

$P(x > 5)$
Given $n = 10, p = 0.8$
$P(x > 5) = p(6) + p(7) + p(8) + p(9) + p(10)$
$= 0.088 + 0.201 + 0.302 + 0.268 + 0.107$
$= 0.966$

Geometric Distribution

Note that:
- Trial means observation
- "success" is a name of one of the possible outcomes result.

Suppose an experiment consists of a sequence of trials with the following conditions:
1. The trails are independent.
2. Each trail can result in either success or failure.
3. The probability of success is the same for all trials.

A **geometric random variable** is defined as

x = number of trails **until the first success** is observed (including the success trail)

Geometric Distribution

Note that:
p = probability of "success".
$(1 - p)$ = probability of "failure".

Outcome	x =Number of trials to First Success	$p(x)$
S	1	$p = (1-p)^0 p$
F S	2	$(1-p)\,p = (1-p)^1 p$
F F S	3	$(1-p)\,(1-p)\,p = (1-p)^2 p$
F F F S	4	$(1-p)\,(1-p)\,(1-p)\,p = (1-p)^3 p$
F F F F S	5	$(1-p)\,(1-p)\,(1-p)(1-p)\,p = (1-p)^4 p$

Geometric Distribution

Geometric Probability Distribution

If x is a geometric random variable with probability of success $= p$ for each trial, then

$$p(x) = (1 - p)^{x-1}p \quad x = 1, 2, 3, \dots$$

Example 4

Example 7.22 (page.380) – Jumper Cables

The scenario described on page.379

x = number of students who must be stopped before finding a student with jumper cables
40% students drive with the jumper cables → probability of "success" → $p = P(S)$

Hence, the probability distribution of x is given by

$$p(x) = (0.6)^{x-1}\,(0.4)\,, \qquad x = 1, 2, \ldots\ldots$$

a. $\boldsymbol{P(x = 1)}$

b. $\boldsymbol{P(x \leq 3)}$

Example 4

Example 7.22 (page.380) – Jumper Cables Cont'

$$p(x) = (0.6)^{x-1}\,(0.4)\,, \qquad x = 1, 2, \ldots\ldots$$

$\boldsymbol{a}$. $\boldsymbol{P(x = 1)}$
$= p(1) = (0.6)^{1-1}\,(0.4)$
$= (0.6)^{0}\,(0.4)$
$= 1\,(0.4)$
$= (0.4)$

$\boldsymbol{b}$. $\boldsymbol{P(x \leq 3)}$
$= p(1) + p(2) + p(3)$
$= (0.6)^{1-1}\,(0.4) + (0.6)^{2-1}\,(0.4) + (0.6)^{3-1}\,(0.4)$
$= 0.4 + 0.24 + 0.144$
$= 0.784$

Conclusion

- Discrete probability distributions
  - Binomial Distribution (Given $n, p(s)$)
    - Formula
      $$p(x) = \frac{n!}{x!\,(n-x)!}\,(p)^{x}(1-p)^{(n-x)}\,, \qquad x = 0, 1, 2, \ldots\ldots, n$$
    - Table – Appendix 9.
  - Geometric Distribution – Until the first success (Given $p(s)$)
    - Formula
      $$p(x) = (1-p)^{x-1}\,(p)\,, \qquad x = 1, 2, \ldots\ldots$$