Univariate Statistical Analysis

Lecture 7

Hypothesis Testing (two tailed)
Chapter 10
Section 10.1, 10.2, 10.4

Today

- Hypotheses and Test Procedures
- Hypothesis Test for Population Mean (Two Tailed)
  - z test
  - t test

Hypotheses and Test Procedures

A hypothesis is a claim or statement about the value of a single population characteristic.

A test of hypotheses is a method that uses sample data to decide between two competing claims (hypotheses) about the population characteristic.

We initially assume that a particular hypothesis, called the **null hypothesis**, denoted by H_0, is the correct one (**the fact**). We then consider the evident (the sample data) and reject the null hypothesis in favor of the completing hypothesis, called the **alternative hypothesis**, denoted by H_1 or H_a only if *convincing* evidence **against the null hypothesis (new idea to test)**.

The rational is we collected the sample data and through the hypothesis test, we can draw a conclusion of whether we should reject H_0 (the fact) or fail to reject H_0.

Hypotheses and Test Procedures

DEFINITIONS

Null hypothesis: A claim about a population characteristic that is initially assumed to be true. The null hypothesis is denoted by H_0.

Alternative hypothesis: A competing claim about a population characteristic. The alternative hypothesis is denoted by H_a.

In carrying out a test of H_0 versus H_a, the null hypothesis H_0 will be rejected in favor of H_a only if the sample provides convincing evidence that H_0 is false.

If the sample does not provide such evidence, H_0 will not be rejected.

The two possible conclusions in a test of hypotheses are *reject* H_0 or *fail to reject* H_0.

Hypotheses Test Population Mean

Steps

1. Define or State H_0 and H_a.
2. Decide one tail or two tails test.
3. Sketch a normal curve.
4. Find the critical value for z or t.
5. Label step 4. result on the curve.
6. Find the test statistic (value)
7. Label step 6. result on the curve.
8. Decide to reject H_0 or failed to reject H_0.

Example 1

The average value is 20, we want to test the average value should not be 20. Given sample mean of 19, sample size of 50, population standard deviation of 2.1 and the level of significant is 0.05.

Summarize the data:

μ = Population Mean = 20

$\bar{x}$ = Sample Mean = 19

σ = Population Standard Deviation = 2.1 (z-test)

n = Number of Samples = 50

$\propto$ = Significant Level = 0.05

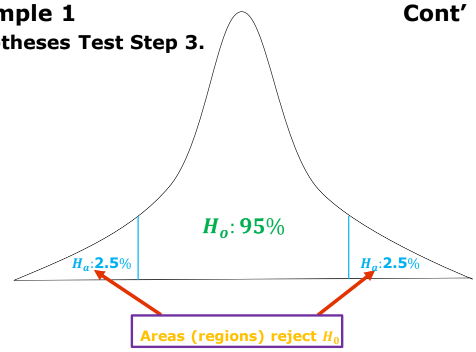
CI $= 1 - \propto = 1 - 0.05 = 0.95 = 95\%$

Example 1 **Cont’**
Hypotheses Test Step 1. and 2.

H_0 : μ = 20

H_a : μ ≠ 20
(≠ → 2 tailed test, so ∝ needs to divide by 2 = 0.025)

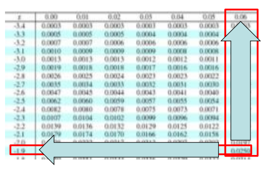
Example 1 **Cont’**
Hypotheses Test Step 3.



Example 1 **Cont’**
Hypotheses Test Step 4.

Critical value $z_{\frac{\propto}{2}}$ (Two tailed)

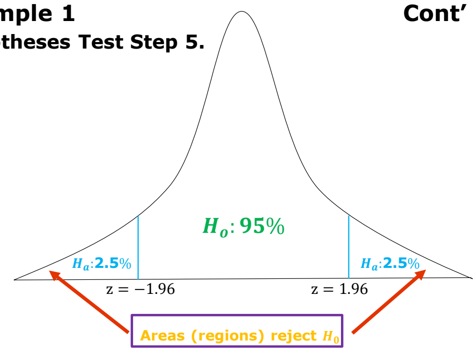
$z_{\frac{0.05}{2}} = z_{0.025} = -1.96$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	[illegible]
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	[illegible]
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	[illegible]
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	[illegible]
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	[illegible]
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	[illegible]
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	[illegible]
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	[illegible]
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	[illegible]
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	[illegible]
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	[illegible]
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	[illegible]
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	[illegible]
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	[illegible]
-2.0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.0197
-1.9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.0250

So, critical values are -1.96 and $+1.96$

Example 1 **Cont’**
Hypotheses Test Step 5.



Example 1 **Cont’**
Hypotheses Test Step 6.

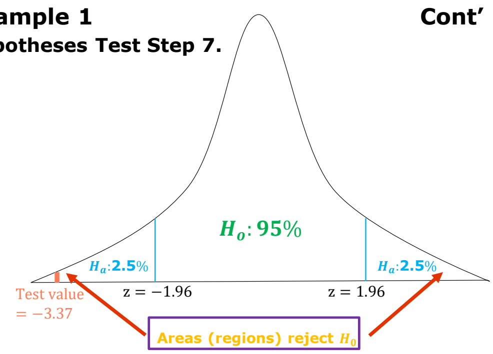
Test value

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{19 - 20}{\frac{2.1}{\sqrt{50}}}$$

$$= \frac{-1}{0.297}$$

$$= -3.37$$

Example 1 **Cont’**
Hypotheses Test Step 7.



Example 1 — Cont'

Hypotheses Test Step 8.

Conclusion:
Reject H_o. There is **enough evidence** to conclude that the average value should not be 20 (H_a).

Example 2

A machine that produces ball with the average diameter 7cm, we want to test the average diameter should not be 7cm. Given sample mean of 7.5cm, sample size of 50, population standard deviation of 2cm and the level of significant is 0.01.

Summarize the data:
μ = Population Mean = 7cm

$\bar{x}$ = Sample Mean = 7.5cm

σ = Population Standard Deviation = 2cm (z-test)

n = Number of Samples = 50

$\propto$ = Significant Level = 0.01

CI $= 1 - \propto = 1 - 0.01 = 0.99 = 99\%$

Example 2 — Cont'

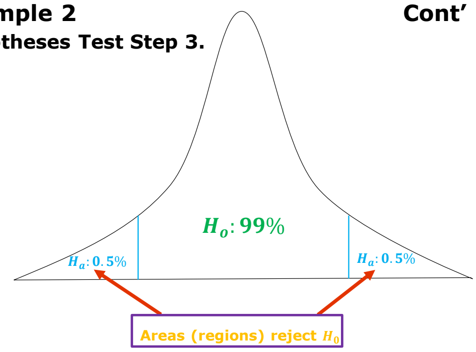
Hypotheses Test Step 1. and 2.

H_0 : μ = 7

H_a : $\mu \neq 7$
($\neq$ → 2 tailed test, so $\propto$ needs to divide by 2 = 0.005)

Example 2 — Cont'

Hypotheses Test Step 3.



Example 2 — Cont'

Hypotheses Test Step 4.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	[illegible]	0.0003
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	[illegible]	[illegible]
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	[illegible]	[illegible]
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	[illegible]	[illegible]
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	[illegible]	[illegible]
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	[illegible]	[illegible]
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	[illegible]	[illegible]
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	[illegible]	[illegible]
-2.6	[illegible]	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	[illegible]	[illegible]
-2.5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.0051	0.0049

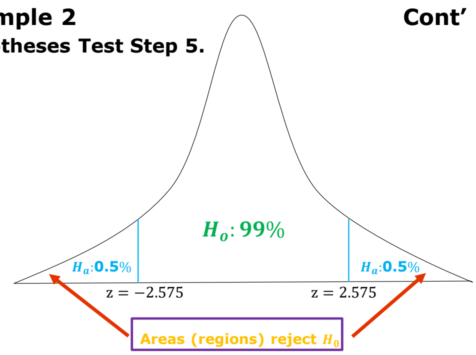
Critical value $z_{\frac{\propto}{2}}$ (Two tailed)

$z_{\frac{0.01}{2}} = z_{0.005} = -2.575$

So, critical values are -2.575 and $+2.575$

Example 2 — Cont'

Hypotheses Test Step 5.



Example 2 Cont’
Hypotheses Test Step 6.

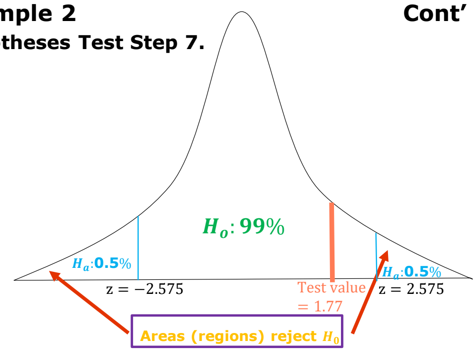
Test value

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{7.5 - 7}{\frac{2}{\sqrt{50}}}$$

$$= \frac{0.5}{0.28}$$

$$= 1.77$$

Example 2 Cont’
Hypotheses Test Step 7.



Example 2 Cont’
Hypotheses Test Step 8.

Conclusion:
Fail to Reject H_o. There is **insufficient evidence** to conclude that the average diameter should not be 7cm (H_a).

Example 3
An article claimed that the mean shopping time at a local supermarket was 30 minutes. Given sample mean of 28.52 minutes, standard deviation of 7.26 minutes and sample of 6 shoppers. Using 0.10 level of significance, can we conclude that mean shopping time is different from the claim?

Summarize the data:

μ = Population Mean = 30 minutes

$\bar{x}$ = Sample Mean = 28.52 minutes

s = Sample Standard Deviation =7.26 minutes (t-test)

n = Number of Samples = 6

∝ = Significant Level = 0.10

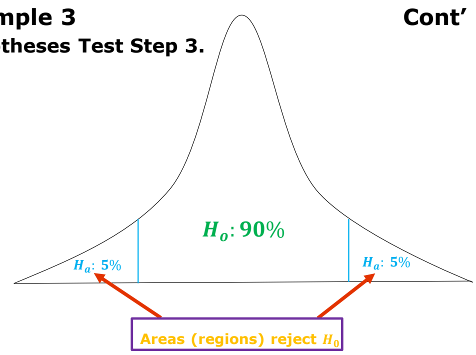
CI = 1− ∝= 1 − 0.10 = 0.90 = 90%

Example 3 Cont’
Hypotheses Test Step 1. and 2.

H_0 : **μ** = 30

H_a : **μ** ≠ 30
(≠ → 2 tailed test, so ∝ needs to divide by 2 = 0.05)

Example 3 Cont’
Hypotheses Test Step 3.



Example 3 Cont’
Hypotheses Test Step 4.

Critical t value

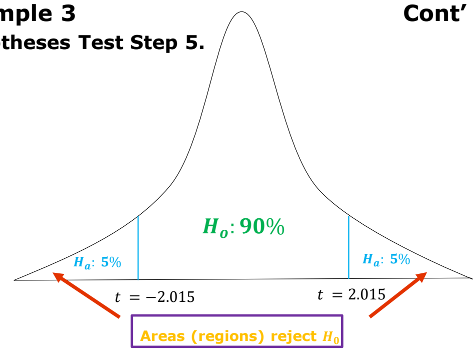
$\propto/2 = 0.05$

$t_{df} = t_{n-1} = t_{6-1} = t_5$

Degrees of freedom	$t_{0.100}$	$t_{0.050}$	$t_{0.025}$	$t_{0.010}$	$t_{0.005}$
1	3.078	[illegible]	12.706	31.821	63.657
2	1.886	[illegible]	4.303	6.965	9.925
3	1.638	[illegible]	3.182	4.541	5.841
4	1.533	[illegible]	2.776	3.747	4.604
5		2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250

So, critical values are -2.015 and $+2.015$ (two tailed)

Example 3 Cont’
Hypotheses Test Step 5.



Example 3 Cont’
Hypotheses Test Step 6.

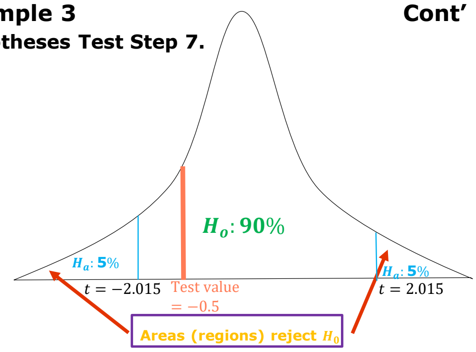
Test value

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{28.52 - 30}{\frac{7.26}{\sqrt{6}}}$$

$$= \frac{-1.48}{2.96}$$

$$= -0.5$$

Example 3 Cont’
Hypotheses Test Step 7.



Example 3 Cont’
Hypotheses Test Step 8.

Conclusion:
Fail to Reject H_o. There is **insufficient evidence** to conclude that the mean shopping time is different from the claim (H_a).

Hypotheses and Test Procedures

DEFINITIONS

Type I error: The error of rejecting H_0 when H_0 is true

Type II error: The error of failing to reject H_0 when H_0 is false

$\propto$ = Significant Level = The probability of Type I error occurs.

DEFINITIONS

The **probability of a Type I error** is denoted by α and is called the **significance level** of the test. For example, a test with $\alpha = 0.01$ is said to have a significance level of 0.01.

The **probability of a Type II error** is denoted by β.

Hypotheses and Test Procedures

	H_o is true	H_o is false
Fail to reject H_o	Correct	Type II error P(Type II error) = β
Reject H_o	Type I error P(Type I error) = ∝	Correct

https://www.youtube.com/watch?v=a_I991xUAOU

Conclusion

- Hypotheses and Test Procedures
- Hypothesis Test for Population Mean, (Two Tailed, ∝/2)
- Steps

1. Define or State
   H_0: μ =
   H_a: μ ≠

2. Decide one tail or two tails test.
 Two Tailed, ∝/2

3. Sketch a normal curve.

Conclusion

4. Find the critical value for z or t.
   Table value

5. Label step 4. result on the curve.

6. Find the test statistic (value)
   - z test (σ is known)

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

   - t test (σ is unknown)

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

7. Label step 6. result on the curve.

Conclusion

8. Decide to reject H_0 or failed to reject H_0.
   - **Reject H_o.** There is **enough evidence** to conclude H_a is true.
   - **Fail to Reject H_o.** There is **insufficient evidence** to conclude that H_a is true.